

# Optimal Condition of Vibration Absorber Used in Calm Water (Comprehensive Study by Quasi-Newton Method)

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## ABSTRACT

In order to obtain the optimal condition for designing a vibration absorber used in calm water, a two-degree-of-freedom system composed of a main vibrating-system and a vibration absorber in the water has been considered. Then, the system has been assumed to vibrate vertically due to the forced displacement applied at its top, and the optimal condition for the absorber has been determined so as to minimize the amplitude of the main system. There are six design-parameters affecting the optimal condition of the absorber in the water instead of four parameters in air. In this study, Quasi-Newton method has been applied to the system to determine the optimal combination of those six design parameters simultaneously. The result indicates that among the six parameters, four parameters greatly affect the performance of absorber in the water within the range of this study.

**KEY WORDS:** Vibration Control in Calm Water, Vibration Absorber, Optimization, Quasi-Newton Method, Six Governing Parameters

## INTRODUCTION

With progress of the ocean development, controls of vibrating bodies in water have been required in many cases, and a vibration absorber could be considered as one of the control systems. Up to the present, the optimal condition for designing a vibration absorber used in air has been extensively studied by many researchers (for example, Den Hartog, 1947; Reed, 1961) and now it is well established. However, the condition for an absorber used in water has not been reported yet in any publications. In the previous study, the authors (Aso and Kobayashi, 1998) found the optimal combination of vibration absorber used in calm water, but only three parameters among the afore-mentioned six design parameters were optimized, keeping the other parameters

constant.

In this study, Quasi-Newton method was applied to the system to determine the optimal combination of the six design parameters simultaneously, and the comprehensive optimal-condition for an absorber in the water has been determined, clarifying the effects of the six design parameters concerned.

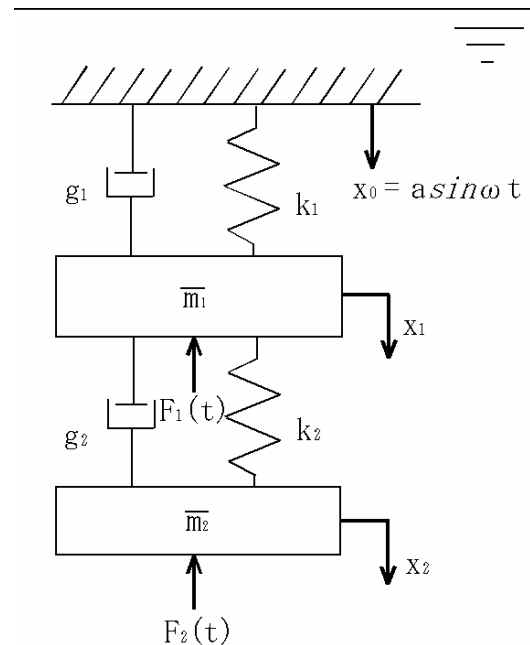


Fig. 1 Schematic diagram of the vibrating system

## ANALYSIS

For examining the effect of the absorber on the vibration of a main system, the vibration of the two-degree-of-freedom system shown in Fig. 1 has been theoretically analyzed. The equations of motion for this system are expressed as follows:

$$\begin{cases} \bar{m}_1 \ddot{x}_1 + (g_1 + g_2) \dot{x}_1 + (k_1 + k_2) x_1 - g_2 \dot{x}_2 \\ - k_2 x_2 + F_1(t) = g_1 \dot{x}_0 + k_1 x_0 \\ \bar{m}_2 \ddot{x}_2 - g_2 \dot{x}_1 - k_2 x_1 + g_2 \dot{x}_2 + k_2 x_2 + F_2(t) = 0 \end{cases} \quad (1)$$

where  $k_i, g_i, \bar{m}_i, F_i(t), x_i$  ( $i=1$  for the main system, 2 for the absorber) are the spring constants, viscous damping coefficients of dampers, masses, and fluid forces due to the ambient water, respectively. Besides,  $x_0$  is the forced displacement at the top of the system, which is assumed as  $a \sin \omega t$  in this study.

The fluid forces,  $F_i(t)$  ( $i=1,2$ ), can be calculated by the following Morison's equation (Morison et al.,1950).

$$F_i(t) = C_{Mi} m_{ai} \ddot{x}_i + 0.5 \rho C_{Di} S_i \dot{x}_i | \dot{x}_i | \quad (i=1,2) \quad (2)$$

where  $C_{Mi}, C_{Di}$  are the added mass and drag coefficients, and where  $m_{ai}, \rho, S_i$  are the masses of the ambient water displaced by  $\bar{m}_i$ , density of the ambient water, cross-sectional area of  $\bar{m}_i$ , respectively. In this analysis, the forces are evaluated by the following modified Morison's equation, which has the drag term linearized by the energy method.

$$\begin{aligned} F_i(t) &\cong C_{Mi} m_{ai} \ddot{x}_i + \left( \frac{4 \rho C_{Di} S_i a_i}{3 \pi} \omega \right) \dot{x}_i \\ &= \tilde{m}_i \ddot{x}_i + c_i \dot{x}_i \quad (i=1,2) \end{aligned} \quad (3)$$

where  $a_i, \omega, \tilde{m}_i, c_i$  are the amplitudes of  $\bar{m}_i$ , and angular frequency, added mass and equivalent damping coefficients, respectively.  $c_i$  are the functions of amplitudes  $a_i$ .

Here, the total masses including their added masses are defined as follows:

$$m_i = \bar{m}_i + \tilde{m}_i \quad (i=1,2) \quad (4)$$

and the following non-dimensional quantities are defined,

$$\left. \begin{aligned} X_1 &= \frac{x_1}{a}, \quad X_2 = \frac{x_2}{a}, \quad T = \omega t \\ \Omega &= \omega \sqrt{\frac{m_1}{k_1}}, \quad \gamma = \frac{k_2}{k_1}, \quad \delta = \frac{g_2}{g_1}, \quad \bar{G}_1 = \frac{g_1}{\sqrt{m_1 k_1}} \\ \mu &= \frac{m_2}{m_1}, \quad \bar{S} = \frac{C_{D2} S_2}{C_{D1} S_1}, \quad C_1 = \frac{c_1}{m_1 \omega}, \quad C_2 = \frac{c_2}{m_2 \omega} \end{aligned} \right\} \quad (5)$$

Then, the non-dimensional equations of motion are written as follows:

$$\begin{aligned} \ddot{X}_1 + \left( \frac{\bar{G}_1}{\Omega} + \frac{\bar{G}_1 \delta}{\Omega} + C_1 \right) \dot{X}_1 + \frac{1}{\Omega^2} (1 + \gamma) X_1 \\ - \frac{\bar{G}_1 \delta}{\Omega} \dot{X}_2 - \frac{\gamma}{\Omega^2} X_2 = \frac{\bar{G}_1}{\Omega} \cos T + \frac{1}{\Omega^2} \sin T \end{aligned}$$

$$\begin{aligned} \ddot{X}_2 - \frac{\bar{G}_1 \delta}{\mu \Omega} \dot{X}_1 - \frac{\gamma}{\mu \Omega^2} X_1 + \left( \frac{\bar{G}_1 \delta}{\mu \Omega} + C_2 \right) \dot{X}_2 \\ + \frac{\gamma}{\mu \Omega^2} X_2 = 0 \end{aligned} \quad (6)$$

Next, the steady-state solution for Eq. 6 can be assumed as the following forms:

$$\left. \begin{aligned} X_1 &= A_1 \cos T + B_1 \sin T, \\ X_2 &= A_2 \cos T + B_2 \sin T \end{aligned} \right\} \quad (7)$$

After substitution of these solutions into Eq. 6, the following simultaneous equations are obtained by equating the cosine and sine terms, respectively.

$$\left. \begin{aligned} (1 + \gamma - \Omega^2) A_1 + \{ \bar{G}_1 \Omega (1 + \delta) + C_1 \Omega^2 \} B_1 \\ - \gamma A_2 - \bar{G}_1 \Omega \delta B_2 = \Omega \bar{G}_1 \\ - \{ \bar{G}_1 \Omega (1 + \delta) + C_1 \Omega^2 \} A_1 + (1 + \gamma - \Omega^2) B_1 \\ + \bar{G}_1 \Omega \delta A_2 - \gamma B_2 = 1 \\ - \gamma A_1 - \bar{G}_1 \Omega \delta B_1 + (\gamma - \Omega^2 \mu) A_2 \\ + \{ \bar{G}_1 \Omega \delta + \mu \Omega^2 C_2 \} B_2 = 0 \\ \bar{G}_1 \Omega \delta A_1 - \gamma B_1 - \{ \bar{G}_1 \Omega \delta + \mu \Omega^2 C_2 \} A_2 \\ + (\gamma - \Omega^2 \mu) B_2 = 0 \end{aligned} \right\} \quad (8)$$

Thus,  $A_1, A_2, B_1, B_2$  can be obtained by solving these equations, and the vibrations of  $\bar{m}_i$  can be determined.

However,  $C_1$  and  $C_2$  are the functions of the vibration amplitudes of  $\bar{m}_i$ . If the amplitudes of  $X_1$  and  $X_2$  are defined as  $\alpha$  and  $\beta$ , respectively,  $\alpha$  and  $\beta$  are expressed in the following forms:

$$\alpha = \sqrt{A_1^2 + B_1^2}, \quad \beta = \sqrt{A_2^2 + B_2^2} \quad (9)$$

Then,  $C_1$  and  $C_2$  can be represented in terms of  $\alpha$  and  $\beta$  as follows:

$$C_1 = C_0 \alpha, \quad C_2 = C_0 \frac{\bar{S}}{\mu} \beta \quad (10)$$

where  $C_0$  is the following non-dimensional damping coefficient.

$$C_0 = \frac{4 \rho C_{D1} S_1 a_1}{3 \pi m_1} \quad (11)$$

Hence, the initial values of  $\alpha$  and  $\beta$  are assumed in advance, and  $A_1, A_2, B_1, B_2$  are obtained by solving Eq. 8 with the relation of Eq. 10 in iterative manner. Then, the amplitude of main vibrating system,  $\alpha$ , is minimized.

## METHOD FOR FINDING THE OPTIMAL ABSORBER

From the analysis mentioned so far, the design parameters are found to be  $\mu, \gamma, \delta, \bar{G}_1, C_0, \bar{S}$  and the last two parameters are the inherent ones in case of the absorber used in calm water. This system includes the damping due to the ambient water besides the damping of

the main vibrating system. Hence, so-called fixed-points theory (Asami et. al., 1995) could not be applied to the system. However, Aso and Kobayashi have clarified in the previous study (1998) that the peak amplitude of frequency response curve becomes minimum when the double peaks appeared in the curve have the same value of  $\alpha$ . For instance, Figs. 2 and 3 show the relationships of  $\alpha$  vs.  $\Omega$ , in cases of various values of  $\mu$  or  $\gamma$  with the constant values of the other design parameters mentioned above. In these figures, it can be found that  $\alpha$  becomes minimum when the double-peak amplitudes of the frequency-response curve (solid line) are equal. Now, the values of the parameters causing the minimum amplitude of the main vibrating system design the optimal absorber, and so the optimal condition of absorber in water could be obtained by finding the parameters which equalize the double-peak amplitudes of the frequency-response curve.

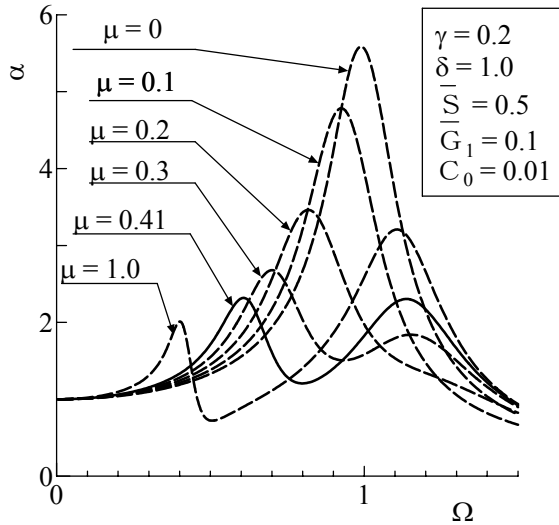


Fig. 2 Frequency response curves for various values of  $\mu$

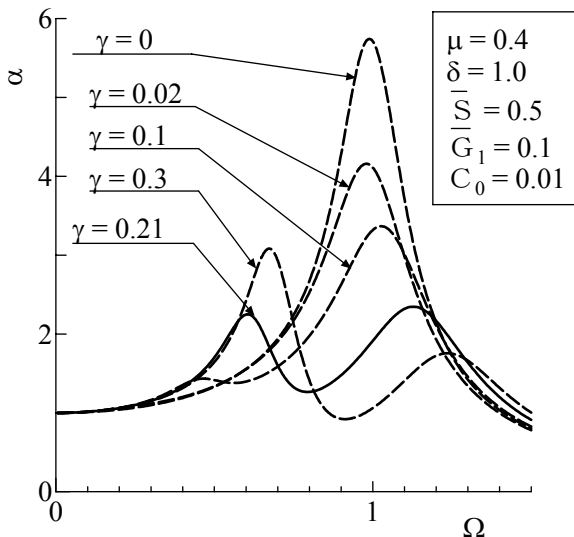


Fig. 3 Frequency response curves for various values of  $\gamma$

In this study, the maximum amplitude of  $\alpha, \alpha'$ , is calculated in advance within the frequency range considered, and then the optimal

combination of the design parameters which minimize  $\alpha'$  is determined by Quasi-Newton method.

Now, the design parameter vector,  $\mathbf{p}$ , is defined as  $\mathbf{p} = (\mu, \gamma, \delta, \bar{G}_1, \bar{S}, C_0)$ , and the maximum amplitude  $\alpha'(\mathbf{p})$  can be obtained from  $\alpha$  as follows:

$$\alpha'(\mathbf{p}) = \max[\alpha(\mathbf{p}, \Omega)]$$

subject to (12)

$$\varepsilon \leq \Omega \leq \Omega_m$$

where  $\varepsilon$  and  $\Omega_m$  are the quantities defining the range of  $\Omega$ . In case of Fig. 2, for example,  $\varepsilon$  and  $\Omega_m$  take 0.01, 1.50, respectively.

Moreover, DFP method (Davidon-Fletcher-Powell method), which is one of the Quasi-Newton methods, is applied to the system to determine the optimal combination of design parameters,  $\mathbf{p}$ , which minimizes the vibration amplitude of the main system. The gradient  $\alpha'_p(\bar{\mathbf{p}})$  and Hessian  $\alpha'_{pp}(\bar{\mathbf{p}})$  for  $\mathbf{p} = \bar{\mathbf{p}}$  are expressed as follows:

$$\alpha'_p(\bar{\mathbf{p}}) = \left[ \frac{\partial \alpha'(\bar{\mathbf{p}})}{\partial p_1}, \frac{\partial \alpha'(\bar{\mathbf{p}})}{\partial p_2}, \dots, \frac{\partial \alpha'(\bar{\mathbf{p}})}{\partial p_n} \right]^T \quad (13)$$

$$\alpha'_{pp}(\bar{\mathbf{p}}) = \begin{bmatrix} \frac{\partial^2 \alpha'(\bar{\mathbf{p}})}{\partial p_1^2} & \frac{\partial^2 \alpha'(\bar{\mathbf{p}})}{\partial p_1 \partial p_2} & \dots & \frac{\partial^2 \alpha'(\bar{\mathbf{p}})}{\partial p_1 \partial p_n} \\ \frac{\partial^2 \alpha'(\bar{\mathbf{p}})}{\partial p_1 \partial p_2} & \vdots & \dots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial^2 \alpha'(\bar{\mathbf{p}})}{\partial p_1 \partial p_n} & \dots & \dots & \frac{\partial^2 \alpha'(\bar{\mathbf{p}})}{\partial p_n^2} \end{bmatrix} \quad (14)$$

where subscripts  $p, pp$  of  $\alpha'_p(\cdot), \alpha'_{pp}(\cdot)$  denote the first and second derivatives with respect to the design parameters,  $p_i (i=1,2, \dots, n)$ , respectively. The bald face characters appearing in the following sentences represent a matrix or a vector.

By applying the DFP method to the system, the approximate expression of Hessian matrix  $\alpha'_{pp}(\bar{\mathbf{p}})$  could be obtained from the gradient vector  $\alpha'_p(\bar{\mathbf{p}})$  as shown below. This method has an advantage when the order of the object system is large and when the objective function is more complicated. Moreover, the gradient,  $\alpha'_p(\mathbf{p})$ , is obtained by the numerical differentiation.

In this study, the inequality-constraint is applied to the design parameters to restrict the range where these parameters could be practical. Hence, in the following calculation algorithm, the modified objective function,  $\bar{\alpha}(\mathbf{p})$ , including the penalty-constraint is minimized instead of the original objective function  $\alpha'(\mathbf{p})$ .

$$\bar{\alpha}(\mathbf{p}) = \alpha'(\mathbf{p}) + r \sum_{i=1}^n \{ \max[0, h_i(\mathbf{p})] \}^2 \quad (15)$$

where  $h_i(\mathbf{p})$  is the function pertaining to the inequality-constraint. When the design parameter vector  $\mathbf{p}$  exists within the practical region,  $h_i(\mathbf{p})$  takes a negative value, otherwise it takes a positive value. Moreover,  $n$  is the number of constraints, and  $r$  is the penalty coefficient. In the following minimization algorithm, the penalty

coefficient,  $r$ , is gradually increased to minimize  $\bar{\alpha}(\mathbf{p})$ .

When  $r$  becomes large enough,  $\alpha'(\mathbf{p})$  is expected to have the minimum value.

The calculation algorithm of the DFP method is expressed as follows (Kanou, 1987):

At the beginning, the inverse-Hessian is defined as  $\mathbf{H}_1$  and  $k = 1$ .

STEP 1) Initial design parameters  $\mathbf{p}_0$  and positive-symmetric matrix  $\mathbf{H}_0$  are assumed.

STEP 2)  $\bar{\alpha}_p(\mathbf{p}_0)$  is calculated, and the procedure is terminated if  $\bar{\alpha}_p(\mathbf{p}_0) \cong 0$ .

STEP 3) The steepest-descend-direction is calculated as  $\mathbf{d}_k = -\mathbf{H}_k \bar{\alpha}_p(\mathbf{p}_k)$ , and  $v_k$  is found to minimize  $\bar{\alpha}(\mathbf{p}_k + v_k \mathbf{d}_k)$  by the line-search method. Then,  $\mathbf{p}_{k+1} = \mathbf{p}_k + v_k \mathbf{d}_k$  is obtained.

STEP 4)  $\bar{\alpha}_p(\mathbf{p}_{k+1})$  is calculated, and the procedure is terminated if  $\bar{\alpha}_p(\mathbf{p}_{k+1}) \cong 0$ .

STEP 5)  $\mathbf{H}_{k+1}$  is obtained by the following DFP equation:

$$\mathbf{H}_{k+1} = \mathbf{H}_k + \frac{\mathbf{s}_k \mathbf{s}_k^T}{\mathbf{y}_k^T \mathbf{s}_k} - \frac{\mathbf{H}_k \mathbf{y}_k \mathbf{y}_k^T \mathbf{H}_k}{\mathbf{y}_k^T \mathbf{H}_k \mathbf{y}_k} \quad (16)$$

where  $\mathbf{s}_k = \mathbf{p}_{k+1} - \mathbf{p}_k$ ,  $\mathbf{y}_k = \bar{\alpha}_p(\mathbf{p}_{k+1}) - \bar{\alpha}_p(\mathbf{p}_k)$ .

STEP 6) The penalty coefficient,  $r$ , is increased as  $r_{k+1} = \lambda r_k$ , ( $\lambda > 1$ ) and  $k = k + 1$ , then the procedure is returned to STEP 3).

The above-mentioned calculation procedure is summarized in the flow chart of Fig. 4.

## RESULT AND DISCUSSIONS

According to the analytical procedure mentioned above, the design parameters are found to be  $\mu, \gamma, \delta, \bar{G}_1, \bar{S}, C_0$ . Here, the inequality constraint is assumed as follows:

$$\begin{cases} \min \bar{\alpha} \\ \text{subject to} \\ 0.01 \leq \mu \leq 1.0 & 0.01 \leq \gamma \leq 1.0 & 0.01 \leq \delta \leq 1.0 \\ 0.01 \leq \bar{G}_1 \leq 0.1 & 0.01 \leq \bar{S} \leq 2.0 & 0.01 \leq C_0 \leq 0.05 \end{cases} \quad (17)$$

The results calculated under the constraint (Eq. 17) for various initial design parameters are shown in Table 1. In these calculations, the initial value of penalty coefficient,  $r_0$ , is selected as 800, and the

incremental ratio,  $\lambda$ , is chosen as 1.2. The convergence histories of the design parameters and the objective function with the initial design parameter,  $\mathbf{p}_0 = (0.2, 0.2, 0.2, 0.05, 0.5, 0.02)$ , are shown in Fig. 5. In this figure, the solid lines indicate the histories of the design parameters and dashed line indicates the history of the objective function  $\bar{\alpha}(\mathbf{p})$ .  $\bar{\alpha}(\mathbf{p})$  decreases with the iteration number, and at 9th iterations it almost converges. After 43rd iterations, the objective function takes the minimum value ( $\bar{\alpha}(\mathbf{p}) = 1.64126$ ). In this case, however, the four parameters except  $\mu$  and  $\gamma$  converge to the upper bounds of the inequality-constraint prescribed in Eq. 17, as shown in Table 1.

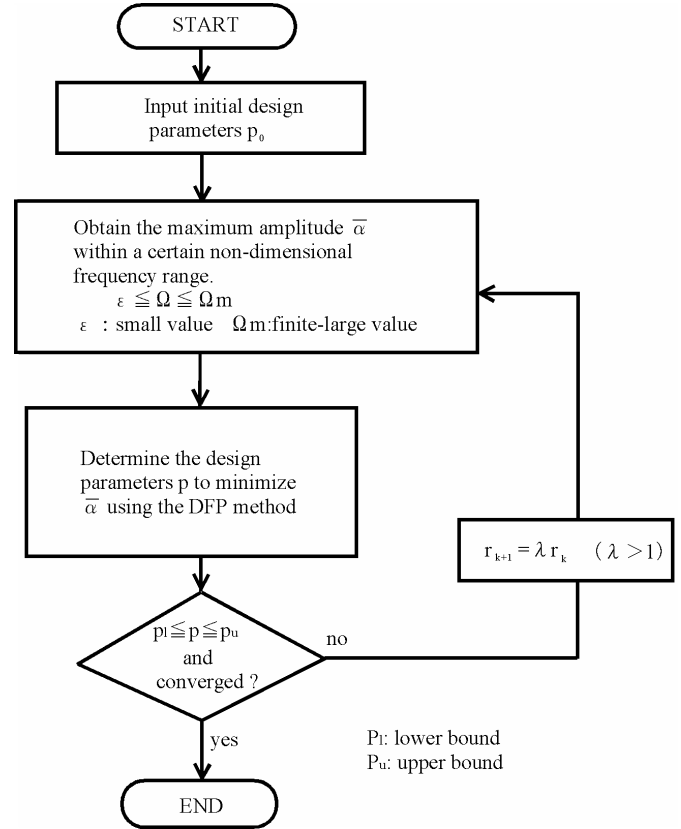


Fig. 4 Flow chart for determining the optimal combination

Table 1 Calculated result in case of the condition prescribed in Eq. 17

		Design Parameter $\mathbf{p} = (\mu, \gamma, \delta, \bar{G}_1, \bar{S}, C_0)$						$\bar{\alpha}$	DFP
Case 1	Initial	0.5	0.5	0.5	0.05	1.0	0.05		
	Converged	<b>0.45485</b>	<b>0.31652</b>	<b>1.0000</b>	<b>0.1000</b>	<b>2.0000</b>	<b>0.0500</b>	<b>1.64126</b>	49
Case 2	Initial	0.2	0.2	0.2	0.05	0.5	0.02		
	Converged	<b>0.45441</b>	<b>0.31636</b>	<b>1.0000</b>	<b>0.1000</b>	<b>1.9999</b>	<b>0.0500</b>	<b>1.64126</b>	43
Case 3	Initial	1.2	1.1	1.3	0.15	2.3	0.06		
	Converged	<b>0.45352</b>	<b>0.31604</b>	<b>0.9999</b>	<b>0.1000</b>	<b>2.0000</b>	<b>0.0500</b>	<b>1.64126</b>	32

Table 2 Calculated result in case of the condition prescribed in Eq. 18

		Design Parameter $\mathbf{p} = (\mu, \gamma, \delta, \bar{G}_1, \bar{S}, C_0)$						$\bar{\alpha}$	DFP
Case 2	Initial	0.2	0.2	0.2	0.05	0.5	0.02		
	Converged	<b>1.66882</b>	<b>0.26559</b>	<b>2.0000</b>	<b>0.2000</b>	<b>4.0000</b>	<b>0.1000</b>	<b>1.21895</b>	24

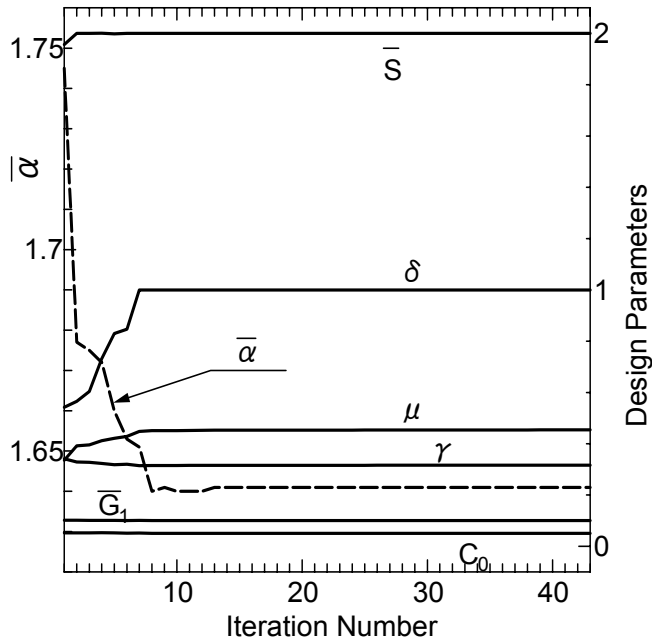


Fig. 5 Convergence histories of the design parameters and the objective function in case 2 (Table 1)

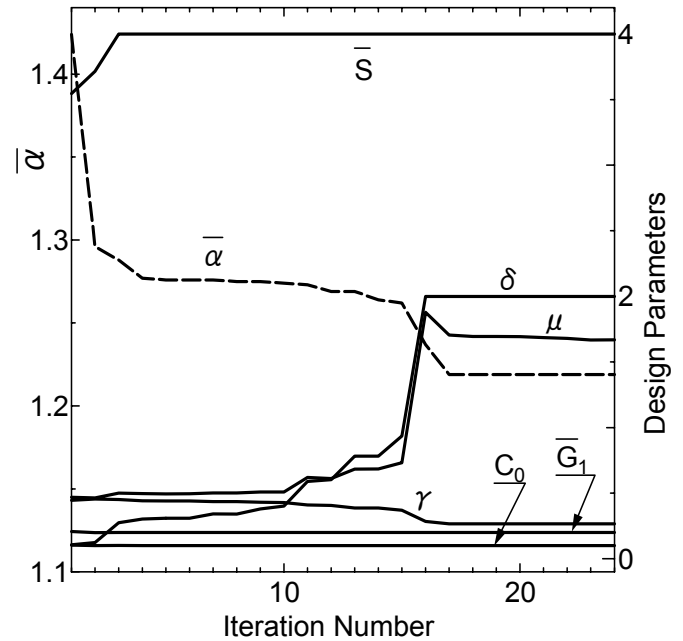


Fig. 6 Convergence histories of the design parameters and the objective function in case 2 (Table 2)

Next, the upper bounds of the inequality-constraint has been extended to two times larger ones than those of the last case, as indicated in Eq. 18. The calculated results and the convergence histories for this case are shown Table 2 and Fig. 6, respectively.

$$\begin{cases} \min \bar{\alpha} \\ \text{subject to} \\ 0.01 \leq \mu \leq 2.0 & 0.01 \leq \gamma \leq 2.0 & 0.01 \leq \delta \leq 2.0 \\ 0.01 \leq \bar{G}_1 \leq 0.2 & 0.01 \leq \bar{S} \leq 4.0 & 0.01 \leq C_0 \leq 0.1 \end{cases} \quad (18)$$

In this case,  $\bar{\alpha}(\mathbf{p})$  considerably decreases at 3rd and 17th iterations, and then the objective function takes the minimum value (1.21895) after 24th iterations.

Fig. 7 indicates the frequency response curves ( $\alpha$  vs.  $\Omega$ ) in cases of the optimal vibration absorbers as well as without absorber, based on the constraints in Eqs. 17 and 18, respectively. Comparison of the two results for the optimal absorbers indicates that the double-peak amplitudes appearing in each frequency response curve are equalized. Hence, the previously mentioned design criterion (Aso and Kobayashi, 1998) based on the equivalent double-peak amplitudes in the frequency-response curve, is verified in this study. Furthermore, it can be seen from Fig. 7 that the resonance frequencies (peak frequencies) for Eq. 18 are shifted to the lower side of frequency, compared with those for Eq.17, and that the peak amplitudes of the main system for Eq.18 are smaller than those for Eq. 17.

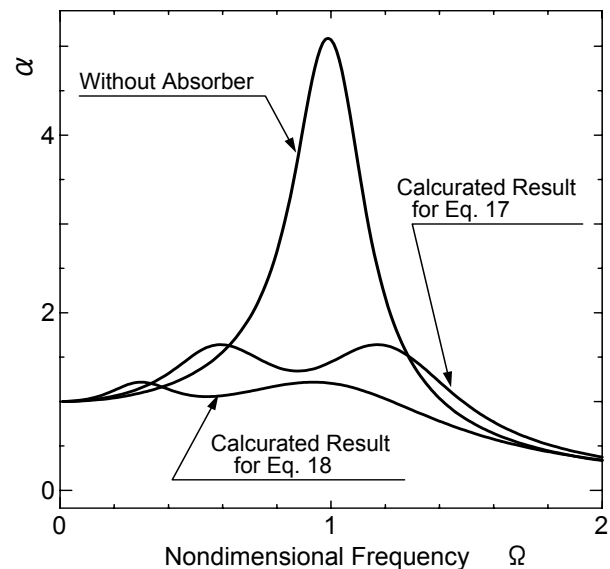


Fig. 7 Frequency response curve after the optimal design

The results of Table 2 as well as Table 1 also indicate that the four parameters except  $\mu$  and  $\gamma$ , converge to the upper bounds of their prescribed ranges, though  $\mu$  and  $\gamma$  take the values within their range.

Table 3 Sensitivity Analysis of parameters to the performance of absorber

		1.0	1.5	2.0	2.5	3.0	3.5
$\delta$	$\delta$	1.000	1.500	1.999	2.500	3.000	3.500
	$\alpha$	1.64126	1.59852	1.55789	1.51881	1.48084	1.45633
	$\Delta\alpha/\Delta\delta$		<b>-0.08548</b>	<b>-0.08345</b>	<b>-0.08163</b>	<b>-0.08021</b>	<b>-0.07397</b>
$\bar{G}_1$	$\bar{G}_1$	0.1000	0.1500	0.2000	0.25000	0.3000	0.35000
	$\alpha$	1.64126	1.54448	1.46115	1.38870	1.32866	1.29500
	$\Delta\alpha/\Delta\bar{G}_1$		<b>-1.9356</b>	<b>-1.8011</b>	<b>-1.6837</b>	<b>-1.5630</b>	<b>-1.3850</b>
$\bar{S}$	$\bar{S}$	1.9999	3.0000	4.0000	5.0000	6.0000	3.50000
	$\alpha$	1.64126	1.56120	1.50222	1.45578	1.41759	1.38530
	$\Delta\alpha/\Delta\bar{S}$		<b>-0.08006</b>	<b>-0.06952</b>	<b>-0.06183</b>	<b>-0.05591</b>	<b>-0.05119</b>
$C_0$	$C_0$	0.050	0.075	0.100	0.1250	0.1500	0.1750
	$\alpha$	1.64126	1.52267	1.44072	1.37949	1.33151	1.29271
	$\Delta\alpha/\Delta C_0$		<b>-4.7436</b>	<b>-4.0108</b>	<b>-3.4903</b>	<b>-3.0975</b>	<b>-2.7884</b>

Hence, it is easily recognized that greater values of  $\delta, \bar{G}_1, \bar{S}, C_0$  are more effective for reducing the amplitude of the main system within the range of this study. Consequently, it is recommended that the greatest values of these four parameters within their allowable ranges are selected in advance, and then,  $\mu$  and  $\gamma$  are optimized to minimize the vibration amplitude of main system. However, those four parameters are usually determined before designing the vibration absorber in order to satisfy the limitations assigned during the design of the main system. Moreover, all of the four parameters could not take the greatest values simultaneously from the practical limitations. Hence, it is necessary to examine the preference order of the four parameters when assigning the value to them.

Then, the sensitivity analysis has been performed to determine the effect of these four parameters on the reduction of  $\alpha$ . Table 3 shows the calculated result when the upper bounds of inequality constraints of the four parameters are extended to 1.5 - 3.5 times larger bounds from those prescribed in Eq. 17. This table indicates the converged values of the parameters, the corresponding values of  $\alpha$  and the sensitivities ( $\Delta\alpha/\Delta\delta, \Delta\alpha/\Delta\bar{G}_1, \Delta\alpha/\Delta\bar{S}, \Delta\alpha/\Delta C_0$ ) based on the values of constraints prescribed in Eq. 17.

Here, it is recognized again that the above-mentioned four parameters always converge to the upper bounds of their inequality constraints. From this table, it is can be found that the effects of  $C_0$  and  $\bar{G}_1$  on the reduction of  $\alpha$  are considerably large, whereas the effects of  $\delta$  and  $\bar{S}$  are very small. Moreover, the influential order of the four parameters on the reduction of  $\alpha$  is clarified as  $C_0, \bar{G}_1, \delta$  and  $\bar{S}$ .

In future, the mutual effects of the parameters on the optimal condition of the absorber will be examined. Moreover, the theoretical results obtained in this study will have to be examined experimentally.

## CONCLUSION

In order to obtain the optimal design parameters for a vibration absorber used in calm water, the vibration of a two-degree-of-freedom

system in the water has been analyzed, and the optimal conditions for the absorber have been determined by the Quasi-Newton method.

The results obtained are as follows:

1. There are six design parameters for an absorber in calm water, and the optimal combination of these parameters could be determined simultaneously by the DFP method (Quasi-Newton method).
2. For designing the optimal absorber in the water, the values of the four parameters  $\delta, \bar{G}_1, \bar{S}, C_0$  among the six parameters should be selected as the largest ones within their allowable ranges, and then,  $\mu$  and  $\gamma$  have to be optimized for minimizing the vibration amplitude of the main system. This finding helps to reduce greatly the labor and time required to design the optimal absorber in water.
3. The effects of the four parameters on the amplitude of the main body are greater in order of  $C_0, \bar{G}_1, \delta$  and  $\bar{S}$ . This influential order (sensitivity) of the parameters also helps to design the optimum absorber when the afore-obtained largest values of the four parameters can not be applied simultaneously from the practical limitations.
4. In future, the method developed in this study will have to be checked widely by experiments, but so far it has been found that the results obtained by this method fairly well coincides with the experimental results within the range of  $\omega$  less than 100 rad/s.

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